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Quiz I MTH 221 , Fall 2019

Basher Husein

Ayman Badawi

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QUESTION 1. Let $Q_1 = (1, 2, 4, -4), Q_2 = (-1, -1, -4, 4), Q_3 = (-2, -4, -7, 8), Q_4 = (2, 4, 8, -8)$. Determine whether $Q_1, ..., Q_4$ are independent or dependent by solving the corresponding homogenous system(i.e., write down the solution set for the corresponding homogeneous system). SHOW THE WORK

$$\alpha_1 Q_1 + \alpha_2 Q_2 + \alpha_3 Q_3 + \alpha_4 Q_4 = (0,0,0,0)$$

$$\alpha_1 (1,2,4,-4) + \alpha_2 (-1,-1,-4,4) + \alpha_3 (-2,-4,-7,8) + \alpha_4 (2,4,8,-8) = (0,0,0,0)$$

$$\alpha_1 - \alpha_2 - 2\alpha_3 + 2\alpha_4 = 0$$
$$2\alpha_1 - \alpha_2 - 4\alpha_3 + 4\alpha_4 = 0$$
$$4\alpha_1 - 4\alpha_2 - 7\alpha_3 + 8\alpha_4 = 0$$
$$-4\alpha_1 + 4\alpha_2 + 8\alpha_3 - 8\alpha_4 = 0$$

(1)
$$\left[\begin{array}{cccc|c} 1 & -1 & -2 & 2 & 0 \\ 2 & -1 & -4 & 4 & 0 \\ 4 & -4 & -7 & 8 & 0 \\ -4 & 4 & 8 & -8 & 0 \end{array}\right] \begin{array}{l} -2R_1 + R_2 \to R_2 \\ -4R_1 + R_3 \to R_3 \\ 4R_1 + R_4 \to R_4 \end{array}$$

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(2)
$$\left[\begin{array}{cccc|c} 1 & -1 & -2 & 2 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad R_2 + R_1 \to R_1$$

(3)
$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 2 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad 2R_3 + R_1 \to R_1$$

(4)
$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

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$\alpha_1 + 2\alpha_4 = 0$

$\alpha_2 = 0$

$\alpha_3 = 0$

α_4 is a free variable

Solution set:
$\{(-2\alpha_4, 0, 0, \alpha_4) \mid \alpha_4 \in \mathbb{R}\}$ 1/1

∴ dependant. 3/3

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Quiz II MTH 221 , Fall 2019

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QUESTION 1. Find the solution set of the following homogeneous system.

$x_1 - x_2 - 4x_3 + x_4 - 2x_5 = 0, \quad -x_1 + x_2 + 4x_3 + x_5 = 0, \quad 2x_1 - 2x_2 - 8x_3 + 3x_4 - 5x_5 = 0$

$$\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \\ \hline 1 & -1 & -4 & 1 & -2 \\ -1 & 1 & 4 & 0 & 1 \\ 2 & -2 & -8 & 3 & -5 \end{array} \xrightarrow[-2R_1+R_3]{R_1+R_2} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \\ \hline 1 & -1 & -4 & 1 & -2 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{array} \xrightarrow[-R_2+R_3]{-R_2+R_1} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \\ \hline 1 & -1 & -4 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

$x_1 - x_2 - 4x_3 - x_5 = 0$

$x_4 - x_5 = 0$

$x_1 = x_2 + 4x_3 + x_5$

$x_4 = x_5$

SS: $\{(x_2+4x_3+x_5, x_2, x_3, x_5, x_5) \mid x_2, x_3, x_5 \in \mathbb{R}\}$

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$x=-1 \quad z=-1$
$y=-1$

QUESTION 2. Convince me that $D = \{(2x + y, x - y, 3z + x + y, -2x) \mid x, y, z \in R\}$ is a subspace of R^4.

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- $(0,0,0,0)$ will always be a solution

$(3,0,5,-2)$
$(-3,0,-5,2)$ $\Rightarrow (0,0,0,0)$

- when we add any 2 points, the point that we get will still be a part of linear variables means D is a subspace of $\mathbb{R}^4$

Also, all variables are linear combination of

QUESTION 3. Is $D = \{(2x - 5y, -y, 3z + 7x + y, 2x) \mid y, z \in R$ and $x \geq 0\}$ a subspace of R^4. Justify your answer?

- $(0,0,0,0)$ is always a solution for this

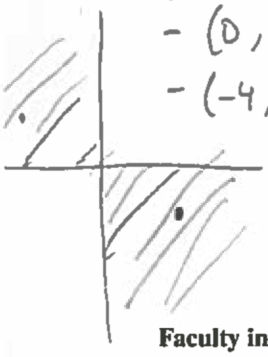
Take: $y=-1$, $z=-1$, $x=1$ $\quad (-3,1,3,2)$ $\quad$ Take: $y=-2$, $z=-2$, $x=2$ $\quad (14,2,6,4)$ $\quad (11,3,9,6)$

if $\alpha = -1$ then the 4th coordinate will be $-$ve so 2nd axiom fails

not a subspace of R^4

3/3

QUESTION 4. Let D be a subset of R^2 that consists of all points in the second quadrant and all points in the fourth quadrent including the x-axis and y-axis. Is D a subspace of R^2? Justify your answer.



- $(0,0)$ is a soln

- $(-4,3) + (6,-2) = \underbrace{(2,1)}$

we get a point that's in the 1st quadrant so the 1st axiom fails hence its not in the subspace of $\mathbb{R}^2$

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Quiz III, MTH 221 , Fall 2019

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QUESTION 1. Let $F = \{(a+3c-d,\ -a+b,\ c+2d,\ 2a-2b,\ 0) \mid a,b,c,d \in R\}$. Then we know that F is a subspace of R^5. Find $IN(F)$(*i.e.*, $dim(F)$) and find a basis for F.

2/2

$$F = \{a(1,-1,0,2,0) + b(0,1,0,-2,0) + c(3,0,1,0,0) + d(-1,0,2,0,0) \mid a,b,c,d \in \mathbb{R}\}$$

$$F = \text{Span}\{(1,-1,0,2,0),\ (0,1,0,-2,0),\ (3,0,1,0,0),\ (-1,0,2,0,0)\}$$

$$\begin{bmatrix} 1 & -1 & 0 & 2 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 3 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 0 \end{bmatrix} \underset{R_1+R_4 \to R_4}{\overset{-3R_1+R_3 \to R_3}{\sim}} \begin{bmatrix} 1 & -1 & 0 & 2 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 3 & 1 & -6 & 0 \\ 0 & -1 & 2 & 2 & 0 \end{bmatrix} \underset{\substack{R_2+R_4 \to R_4 \\ -3R_2+R_3 \to R_3}}{\overset{R_2+R_1 \to R_1}{\sim}}$$

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$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \end{bmatrix} \underset{-2R_3+R_4 \to R_4}{\sim} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

independent

dependent

$$= \text{Span}\{(1,0,0,0,0),\ (0,1,0,-2,0),\ (0,0,1,0,0)\} \quad \& \quad \boxed{IN(F) = 3}$$

2/2

Basis of F = $B = \{(1,0,0,0,0),\ (0,1,0,-2,0),\ (0,0,1,0,0)\}$

QUESTION 2. Let $F = \{(a+2b,\ -c+3b,\ 2a+4b,\ w) \mid w - 6b + 2c = 0$ and $a,b,c \in R\}$. Then we know that F is a subspace of R^4. Find $IN(F)$(*i.e.*, $dim(F)$) and find a basis for F.

$$F = \{(a+2b,\ -c+3b,\ 2a+4b,\ 6b-2c) \mid a,b,c \in \mathbb{R}\}$$

$$F = \{a(1,0,2,0) + b(2,3,4,6) + c(0,-1,0,-2) \mid a,b,c \in \mathbb{R}\}$$

$$F = \text{span}\{(1,0,2,0),\ (2,3,4,6),\ (0,-1,0,-2)\}$$

$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 2 & 3 & 4 & 6 \\ 0 & -1 & 0 & -2 \end{bmatrix} \overset{-2R_1+R_2 \to R_2}{\sim} \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 3 & 0 & 6 \\ 0 & -1 & 0 & -2 \end{bmatrix} \overset{\frac{1}{3}R_2}{\sim} \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & -1 & 0 & -2 \end{bmatrix}$$

$$\underset{R_2+R_3 \to R_3}{\sim} \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

independent

dependent

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$$F = \text{span}\{(1,0,2,0),\ (0,1,0,2)\}$$

$IN(F) = 2$ & $\boxed{\text{Basis of } F = B = \{(1,0,2,0),\ (0,1,0,2)\}}$

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Quiz IV , MTH 221 , Fall 2019

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QUESTION 1. Find the solution set of the following homogeneous system, call it F. Write F as a span of independent points. Then find an orthogonal basis for F.

$$x_1 + x_2 + x_4 + x_5 = 0$$
$$-x_1 - x_2 + x_3 - 2x_4 - 2x_5 = 0$$
$$-x_1 - x_2 - 2x_3 + x_4 + x_5 = 0$$

Augmented matrix

3×5 homogenous system

$$\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array}$$
$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 1 & 1 & 0 \\ -1 & -1 & 1 & -2 & -2 & 0 \\ -1 & -1 & -2 & 1 & 1 & 0 \end{array}\right] \xrightarrow[R_1+R_3 \to R_3]{R_1+R_2 \to R_2} \left[\begin{array}{ccccc} 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & -2 & 2 & 2 \end{array}\right]$$

$$\xrightarrow{2R_2+R_3 \to R_3} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \left[\begin{array}{ccccc} \boxed{1} & 1 & 0 & 1 & 1 \\ 0 & 0 & \boxed{1} & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array} \Rightarrow$$

x_1 & x_3 ⇒ leading variables
x_2 & x_4 & x_5 ⇒ free variables

$$x_1 + x_2 + x_4 + x_5 = 0 \Rightarrow x_1 = -x_2 - x_4 - x_5$$
$$x_3 - x_4 - x_5 = 0 \Rightarrow x_3 = x_4 + x_5$$

$x_2, x_4, x_5 \in \mathbb{R}$

Solution set $= F = \{(-x_2 - x_4 - x_5,\ x_2,\ x_4 + x_5,\ x_4,\ x_5) \mid x_2, x_4, x_5 \in \mathbb{R}\}$

$F = \{x_2(-1,1,0,0,0) + x_4(-1,0,1,1,0) + x_5(-1,0,1,0,1) \mid x_2, x_4, x_5 \in \mathbb{R}\}$

independent

$F = \text{Span}\{(-1,1,0,0,0), (-1,0,1,1,0), (-1,0,1,0,1)\}$

↑
solution set of homogenous system of LE

Basis for $F = B = \{\underbrace{(-1,1,0,0,0)}_{Q_1}, \underbrace{(-1,0,1,1,0)}_{Q_2}, \underbrace{(-1,0,1,0,1)}_{Q_3}\}$

Orthogonal basis for $B = B' = \{W_1, W_2, W_3\}$

$$\boxed{W_1 = Q_1 = (-1,1,0,0,0)}$$

$$\boxed{W_2 = Q_2 - \frac{Q_2 \cdot W_1}{\|W_1\|^2} W_1}$$

$$\boxed{W_3 = Q_3 - \frac{Q_3 \cdot W_2}{\|W_2\|^2} W_2 - \frac{Q_3 \cdot W_1}{\|W_1\|^2} W_1}$$

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$$W_2 = Q_2 - \boxed{\frac{Q_2 \cdot W_1}{\|W_1\|^2}} W_1$$

$$\frac{Q_2 \cdot W_1}{\|W_1\|^2} = \frac{(-1,0,1,1,0)\cdot(-1,1,0,0,0)}{\left(\sqrt{1^2+1^2}\right)^2} = \frac{1+0+0+0+0}{2} = \boxed{\frac{1}{2}}$$

$$W_2 = \left[(-1,0,1,1,0) - \frac{1}{2}(-1,1,0,0,0)\right] \times 2$$

$$= (-2,0,2,2,0) - (-1,1,0,0,0)$$

$$= (-1,-1,2,2,0)$$

$$\boxed{B' = \{(-1,1,0,0,0),\ (-1,-1,2,2,0),\ W_3\}}$$

Khalifa

Quiz V , MTH 221 , Fall 2019

Ayman Badawi

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$n = 4$

$\alpha = 3$

QUESTION 1. Let $A = \begin{bmatrix} 2 & -2 & -3 & -4 \\ 1 & 5 & 3 & 4 \\ 2 & 4 & 9 & 8 \\ 1 & 2 & 2 & 9 \end{bmatrix}$. Is 3 an eiginvalue of A? if yes, then find E_3. Find $IN(E_3)$ (i.e., $dim(E_3)$). Write E_3 as span of orthogonal basis.(you may finish on the back)

$$(\alpha I_4 - A)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left(3\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -2 & -3 & -4 \\ 1 & 5 & 3 & 4 \\ 2 & 4 & 9 & 8 \\ 1 & 2 & 2 & 9 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & -2 & -3 & -4 \\ -2 & -4 & -6 & -8 \\ -1 & -2 & -2 & -6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Augmented matrix $\to$

$$\begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ -1 & -2 & -3 & -4 & 0 \\ -2 & -4 & -6 & -8 & 0 \\ -1 & -2 & -2 & -6 & 0 \end{array}\right] \end{array}$$

$R_1 + R_2 \to R_2$
$-R_1 + R_3 \to R_3$
$R_1 + R_4 \to R_4$

$$\begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \left[\begin{array}{cccc|c} \boxed{1} & 2 & 3 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \boxed{1} & -2 & 0 \end{array}\right] \end{array}$$

$\boxed{x_1} + 2x_2 + 3\boxed{x_3} + 4x_4 = 0$

$\boxed{x_3} - 2x_4 = 0$

leading variables $\to x_1, x_3$

free variables $\to x_2, x_4$

$\boxed{x_3 = 2x_4}$

$x_1 = -2x_2 - 3x_3 - 4x_4$

$x_1 = -2x_2 - 6x_4 - 4x_4$

$\boxed{x_1 = -2x_2 - 10x_4}$ $\quad \boxed{x_2, x_4 \in \mathbb{R}}$

Solution set $= E_3 = \{(-2x_2 - 10x_4, x_2, 2x_4, x_4) \mid x_2, x_4 \in \mathbb{R}\}$

infinite solutions $\Rightarrow \alpha = 3$ is an eiginvalue of A

$IN(E_3) = 2 \leftarrow$ # free variables

QUESTION 2. Let $A = \begin{bmatrix} 2 & -2 & -3 & -4 \\ 1 & 5 & 3 & 4 \\ 2 & 4 & 9 & 8 \\ 1 & 2 & 2 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & 1 & 0 & 2 \\ 1 & 5 & 3 & 2 & 0 \\ 2 & 4 & 9 & 1 & 0 \\ 1 & 2 & 2 & 1 & 2 \end{bmatrix}$. Find

$AB = C$

(i) C_5

$$2\begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix} + 0\begin{bmatrix} -2 \\ 5 \\ 4 \\ 2 \end{bmatrix} + 0\begin{bmatrix} -3 \\ 3 \\ 9 \\ 2 \end{bmatrix} + 2\begin{bmatrix} -4 \\ 4 \\ 8 \\ 9 \end{bmatrix} = \begin{bmatrix} -4 \\ 10 \\ 20 \\ 20 \end{bmatrix} = C_5$$

$\to 0$ $\quad \to 0$

(ii) $c_{2,4}$.

2nd row & 4th column

$c_{2,4} = {}_2A \cdot B_4 = (1, 5, 3, 4)\cdot(0, 2, 1, 1)$

$= 0 + 10 + 3 + 4 = 17$

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$\boxed{c_{2,4} = 17}$

Omar Saqf Elhait

Q1] Find $|A|$ if $A = \begin{bmatrix} 3 & 7 \\ -2 & 5 \end{bmatrix}$

$|A| = (5 \cdot 3) - (7 \cdot -2) = 15 + 14 = 29$

Q2] Find $|A|$ using definition if $A = \begin{bmatrix} 2 & -3 & 4 \\ -1 & 5 & 3 \\ -2 & 0 & 5 \end{bmatrix}$ then $|-4A|$

Here we use ~~see~~ 2nd column

$|A| = (-1)^{1+2} (-3) \begin{vmatrix} -1 & 3 \\ -2 & 5 \end{vmatrix} + (-1)^{2+2} (5) \begin{vmatrix} 2 & 4 \\ -2 & 5 \end{vmatrix} + (-1)^{3+2} (0)$

$|A| = (-1)(-3) \Rightarrow 3\left(((-1 \cdot 5) - (3 \cdot -2))\right) + 5\left((2 \cdot 5) - (4 \cdot -2)\right)$

$= 3(1) + 5(18)$

$|A| = 93$

$|-4A| = (-4)^3 \times 93 = \cancel{}\ -5952$

Q3] $A \xrightarrow{3R_3} B \xrightarrow{-3R_3 + R_4 \to R_4} \overbrace{\begin{bmatrix} 1 & 3 & 4 & -1 \\ 0 & 0 & 3 & 4 \\ 0 & 4 & 4 & 4 \\ 0 & 0 & 0 & 7 \end{bmatrix}}^{C}$ Find $|A|$ and $|B|$ and $|-2A|$

from given $\Rightarrow 3|A| = |B| = C$

$C \xrightarrow{R_2 \leftrightarrow R_3} D \begin{vmatrix} 1 & 3 & 4 & -1 \\ 0 & 4 & 4 & 4 \\ 0 & 0 & 3 & 4 \\ 0 & 0 & 0 & 7 \end{vmatrix}$

$|D| = -|C|$

$|D| = (1 \cdot 4 \cdot 3 \cdot 7) = 84$

$|C| = -84$

$|B| = -84$

$|A| = \frac{-84}{3}$

$|-2A| = (-2)^4 \left(\frac{-84}{3}\right) = \cancel{}\ 448$

Q4] $A = \begin{bmatrix} 4 & -2 & 7 & -1 \\ -8 & 4 & K & 1 \\ -4 & a & b & c \\ -4 & 2 & -7 & m \end{bmatrix}$ find a, b, c, K, m so that $A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ has infinitely many solution

B $\quad -|B| = |C|$

$2R_1 + R_2$
$R_1 + R_3$
$R_1 + R_4$

$$\begin{bmatrix} 4 & -2 & 7 & -1 \\ 0 & 0 & 14+K & -1 \\ 0 & -2+a & 7+b & c-1 \\ 0 & 0 & 0 & m-1 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 4 & -2 & 7 & -1 \\ 0 & a-2 & 7+b & c-1 \\ 0 & 0 & 14+K & -1 \\ 0 & 0 & 0 & m-1 \end{bmatrix}$$

so that it has many solutions

$$4(a-2)(14+K)(m-1) = 0$$

$a - 2 = 0$

$\boxed{a = 2}$ $\quad 14 + K = 0 \quad m - 1 = 0$

$\boxed{K = -14} \quad \boxed{m = 1}$

$a = 2,\ K = -14,\ m = 1,\ b, c \in \mathbb{R}$